

Massive Gauge Bosons in Yang-Mills Theory without Higgs Mechanism

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A new mechanism giving the massive gauge bosons in Yang-Mills theory is proposed in this letter. The masses of intermediate vector bosons can be automatically given without introducing Higgs scalar boson. Furthermore the relation between the masses of bosons and the fermion mass matrix is obtained. It is discussed that the theory should be renormalizable.

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Half a century ago, Yang and Mills constructed the gauge field theory of non-Abelian group, which has become the most fundamental content in modern physics. Upon the principle that physical laws should be covariant under the local isospin rotation they proposed the $SU(2)$ Yang-Mills theory [1]. But they could not obtain the massive gauge bosons then. About 10 years later, An ingenious trick called the Higgs mechanism was independently invented by Higgs and Englert and Brout [2], who introduced a scalar field and the spontaneous symmetry broken mechanism of vacuum by fixing a vacuum expectation value of the scalar field and make the intermediate vector bosons obtain masses. Based on the Yang-Mills fields and the Higgs mechanism, Glashow, Salam and Weinberg *etc.* proposed a renormalizable theory unifying the weak and electromagnetic interactions, namely $SU(2)_L \times U(1)_Y$ gauge theory [3]. In this theory, the neutrinos are assumed to be massless and other fermions can acquire the masses through the Yukawa couplings with the Higgs boson.

Although the $SU(2)_L \times U(1)_Y$ electroweak theory has predicted the masses of intermediate vector bosons, which are confirmed by experiments, there are still several unconfirmed predictions or conflicting phenomena in it. e.g. firstly, experimenters do not find any hints of the Higgs boson till now; secondly, several recent experiments imply that the neutrinos should be massive and be mixed [4]. The neutrino experiments demonstrate that people should modify the $SU(2)_L \times U(1)_Y$ electroweak theory to be consistent with more experimental phenomena. In this letter we propose a mechanism to give the massive gauge bosons in Yang-Mills theory without introducing a scalar boson. The equations connecting the masses of intermediate vector bosons and the fermion mass matrix are also obtained.

We demonstrate our mechanism under the primary framework of Yang-mills theory in this letter. Let's consider a quantum field system in which the fermions $\psi_1(x)$ and $\psi_2(x)$ form an isospin doublet as follows

$$\psi = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}. \quad (1)$$

While ignoring the helicity of fermions, the largest inner gauge symmetry group in this system is obviously

$SU(2) \times U(1)$ group. The complete Lagrangian of this system without mass terms reads

$$\mathcal{L} = i\bar{\psi}\gamma^\mu(\partial_\mu - ig\mathbf{T} \cdot \mathbf{A}_\mu)\psi + g'\bar{\psi}\gamma^\mu A_\mu^0\psi - \frac{1}{4}\mathbf{F}'_{\mu\nu} \cdot \mathbf{F}'^{\mu\nu} - \frac{1}{4}E'_{\mu\nu}E'^{\mu\nu}. \quad (2)$$

Where g, g' are the coupling constants of $SU(2)$ gauge field and $U(1)$ gauge field respectively, and the dot “ \cdot ” denotes a scalar product in the isospace. In this case, $\mathbf{T} \cdot \mathbf{A}_\mu$ means $T_1 A_\mu^1 + T_2 A_\mu^2 + T_3 A_\mu^3$, where T_i are the generators of $SU(2)$ group. In Yang-Mills theory, $\mathbf{T} \cdot \mathbf{A}_\mu$ is called the $SU(2)$ gauge field, and the $F'_{\mu\nu} = \mathbf{F}'_{\mu\nu} \cdot \mathbf{T} = \partial_\mu(\mathbf{A}_\nu \cdot \mathbf{T}) - \partial_\nu(\mathbf{A}_\mu \cdot \mathbf{T}) - ig[\mathbf{A}_\mu \cdot \mathbf{T}, \mathbf{A}_\nu \cdot \mathbf{T}]$ is its field strength. Hence the field strength $\mathbf{F}'_{\mu\nu}$ satisfies

$$\mathbf{F}'_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + g\mathbf{A}_\mu \times \mathbf{A}_\nu. \quad (3)$$

We use the A_μ^0 denote the $U(1)$ gauge field. Its gauge field strength can be expressed as follows

$$E'_{\mu\nu} = \partial_\mu A_\nu^0 - \partial_\nu A_\mu^0. \quad (4)$$

In quantum theory Pauli matrices are usually represented as the following

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5)$$

The generators of $SU(2)$ Lie algebra can be then written as

$$T^i = \frac{1}{2}\tau^i. \quad (6)$$

Furthermore the T^i obey the commutation relations of the $SU(2)$ Lie algebra

$$[T^i, T^j] = i \sum_{k=1}^3 \varepsilon_{ijk} T^k. \quad (7)$$

Here ε_{ijk} is the totally antisymmetry tensor in 3-dimensions. The spherical components of \mathbf{T} are

$$T_+ = T_1 + iT_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad (8)$$

$$T_- = T_1 - iT_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad (9)$$

$$T_3 = \frac{1}{2}\tau_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (10)$$

Therefore the scalar product $\mathbf{A}_\mu \cdot \mathbf{T}$ in the isospace can be expressed in terms of the spherical components of \mathbf{T} , e.g. $\mathbf{A}_\mu \cdot \mathbf{T} = \frac{1}{\sqrt{2}}(A_\mu^+ T_+ + A_\mu^- T_-) + A_\mu^3 T_3$, where the A_μ^+ and A_μ^- are defined by

$$A_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2), \quad (11)$$

The complete Lagrangian density without mass terms are gauge invariant and thus will lead to a renormalizable quantum field theory. But the Lagrangian density of Eq.(2) can not describe the real physical world. To make the vector bosons in $SU(2)$ gauge field obtain the masses, we introduce the transformation

$$A_\mu^i = B_\mu^i - C_\mu^i, \quad i = 0, 1, 2, 3. \quad (12)$$

Where C_μ^i are four special vectors since every components of them are 4×4 matrices. Thus every components of the vector fields B_μ^i must be 4×4 matrices either. Inserting Eq.(12) in Eq.(11) directly yields

$$B_\mu^\pm = \frac{1}{\sqrt{2}}(B_\mu^1 \mp iB_\mu^2), \quad C_\mu^\pm = \frac{1}{\sqrt{2}}(C_\mu^1 \mp iC_\mu^2), \quad (13)$$

and

$$B_\mu^+ T_+ + B_\mu^- T_- = \sqrt{2}(B_\mu^1 T_1 + B_\mu^2 T_2). \quad (14)$$

Here we assume that C_μ^i can be expressed as follows

$$\begin{aligned} C_\mu^- &= \frac{1}{2\sqrt{2}g} m_{21} \gamma_\mu, \quad C_\mu^0 = \frac{1}{8g'} (m_{11} + m_{22}) \gamma_\mu, \\ C_\mu^+ &= \frac{1}{2\sqrt{2}g} m_{12} \gamma_\mu, \quad C_\mu^3 = \frac{1}{4g} (m_{11} - m_{22}) \gamma_\mu. \end{aligned} \quad (15)$$

Where m_{11} , m_{22} are real parameters, and m_{12} , m_{21} satisfy $m_{12}^* = m_{21}$ (here * denotes complex conjugate).

Inserting the equations (12), (13), (14) and (15) in Eq.(2), and using the identity, e.g. $\gamma^\mu \gamma_\mu = 4$, one can rewrite the Lagrangian density of Eq.(2) as follows

$$\begin{aligned} \mathcal{L} &= i\bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{g}{\sqrt{2}} \bar{\psi} \gamma^\mu B_\mu^+ T_+ \psi \\ &+ \frac{g}{\sqrt{2}} \bar{\psi} \gamma^\mu B_\mu^- T_- \psi + g \bar{\psi} \gamma^\mu B_\mu^3 T_3 \psi \\ &+ g' \bar{\psi} \gamma^\mu B_\mu^0 \psi - \bar{\psi} \hat{M} \psi \\ &- \frac{1}{4} \mathbf{F}'_{\mu\nu} \cdot \mathbf{F}'^{\mu\nu} - \frac{1}{4} E'_{\mu\nu} E'^{\mu\nu}. \end{aligned} \quad (16)$$

Where the operator

$$\hat{M} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \quad (17)$$

is obviously a Hermitian operator, namely, $\hat{M}^\dagger = \hat{M}$. The term $-\bar{\psi} \hat{M} \psi$ in Eq.(16) can be looked as the mass

term of the fermion fields. Hence the operator \hat{M} is the fermion mass matrix.

In Yang-Mills theory the isospin doublet ψ can be transformed by

$$\psi \rightarrow \tilde{\psi} = U\psi = \exp(i\mathbf{a}(x) \cdot \mathbf{T})\psi. \quad (18)$$

Under the transformation Eq.(18) of the isospin doublet, the $SU(2)$ gauge field $\mathbf{A}_\mu \cdot \mathbf{T}$ transforms as follows

$$\mathbf{A}'_\mu \cdot \mathbf{T} = U \mathbf{A}_\mu \cdot \mathbf{T} U^{-1} + \frac{i}{g} U (\partial_\mu U^{-1}). \quad (19)$$

After we do the transformation of Eq.(12), the invariant requirement of the lagrangian of Eq.(2) under the local $SU(2) \times U(1)$ gauge transformations makes sure that

$$\mathbf{C}'_\mu \cdot \mathbf{T} = U \mathbf{C}_\mu \cdot \mathbf{T} U^{-1}. \quad (20)$$

Inserting Eq.(12) in Eq.(19) and minus Eq.(20) then yields

$$\mathbf{B}'_\mu \cdot \mathbf{T} = U \mathbf{B}_\mu \cdot \mathbf{T} U^{-1} + \frac{i}{g} U (\partial_\mu U^{-1}). \quad (21)$$

It is intriguing that the \mathbf{B}_μ is also an $SU(2)$ Yang-Mills field.

Following the same sequence as the above argument on the local $SU(2)$ gauge transformations, one can similarly prove that B_μ^0 should be treated as a $U(1)$ gauge field provided that the transformation matrix $U = \exp(i\mathbf{a}(x) \cdot \mathbf{T})$ is substituted by the phase factor $U = \exp(i\theta(x))$ in Eq.(18) and the C_μ^0 transforms as

$$(C_\mu^0)' = C_\mu^0 \quad (22)$$

under the local $U(1)$ rotation. Hence one can define the field strength of B_μ^0 field as follows

$$E_{\mu\nu} = \partial_\mu B_\nu^0 - \partial_\nu B_\mu^0. \quad (23)$$

Inserting Eq.(12) and Eq.(15) in Eq.(4) directly yields $E'_{\mu\nu} = \partial_\mu B_\nu^0 - \partial_\nu B_\mu^0$. That is to say,

$$E_{\mu\nu} = E'_{\mu\nu}. \quad (24)$$

Through Eq.(16) and Eq.(24), we can draw a conclusion that the B_μ^0 is a massless vector field.

In complete Lagrangian of Eq.(16), the energy density of the $SU(2)$ gauge field is

$$\mathcal{L}_{SU(2)} = -\frac{1}{4} \mathbf{F}'_{\mu\nu} \cdot \mathbf{F}'^{\mu\nu}. \quad (25)$$

In the following we will consider the change of the $\mathcal{L}_{SU(2)}$ under the transformation from \mathbf{A}_μ to \mathbf{B}_μ . According to the equations (3), (12), (13) and (14), we rewrite the gauge field energy density of Eq.(25) as[6]

$$\begin{aligned}
\mathbf{F}'_{\mu\nu} \cdot \mathbf{F}'^{\mu\nu} &= \sum_{i=1}^3 [\partial_\mu B_\nu^i - \partial_\nu B_\mu^i + g\varepsilon_{ikl}(B_\mu^k - C_\mu^k)(B_\nu^l - C_\nu^l)] \times \\
&[\partial^\mu B^{i\nu} - \partial^\nu B^{i\mu} + g\varepsilon_{imn}(B^{m\mu} - C^{m\mu})(B^{n\nu} - C^{n\nu})] \\
&= 2[(\partial_\mu B_\nu^- - \partial_\nu B_\mu^-) - ig(B_\mu^- B_\nu^3 - B_\nu^- B_\mu^3) - igV_{\mu\nu}] \times \\
&[(\partial^\mu B^{+\nu} - \partial^\nu B^{+\mu}) + ig(B^{+\mu} B^{3\nu} - B^{+\nu} B^{3\mu}) + igX^{\mu\nu}] + \\
&[(\partial_\mu B_\nu^3 - \partial_\nu B_\mu^3) + ig(B_\mu^- B_\nu^+ - B_\nu^- B_\mu^+) + igY_{\mu\nu}] \times \\
&[(\partial^\mu B^{3\nu} - \partial^\nu B^{3\mu}) + ig(B^{-\mu} B^{+\nu} - B^{-\nu} B^{+\mu}) + igZ^{\mu\nu}] \\
&= \mathbf{F}_{\mu\nu} \cdot \mathbf{F}^{\mu\nu} + 2ig(\partial_\mu B_\nu^- - \partial_\nu B_\mu^-)X^{\mu\nu} + 2g^2(B_\mu^- B_\nu^3 - B_\nu^- B_\mu^3)X^{\mu\nu} \\
&- 2ig(\partial^\mu B^{+\nu} - \partial^\nu B^{+\mu})V_{\mu\nu} + 2g^2(B^{+\mu} B^{3\nu} - B^{+\nu} B^{3\mu})V_{\mu\nu} + 2g^2V_{\mu\nu}X^{\mu\nu} \\
&+ ig(\partial_\mu B_\nu^3 - \partial_\nu B_\mu^3)Z^{\mu\nu} - g^2(B_\mu^- B_\nu^+ - B_\nu^- B_\mu^+)Z^{\mu\nu} - g^2Y_{\mu\nu}Z^{\mu\nu} \\
&+ ig(\partial^\mu B^{3\nu} - \partial^\nu B^{3\mu})Y_{\mu\nu} - g^2(B^{-\mu} B^{+\nu} - B^{-\nu} B^{+\mu})Y_{\mu\nu} .
\end{aligned} \tag{26}$$

Where

$$\mathbf{F}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu + g\mathbf{B}_\mu \times \mathbf{B}_\nu . \tag{27}$$

The Yang-Mills theory shows that the $\mathbf{F}_{\mu\nu}$ is the field strength of the $SU(2)$ gauge field \mathbf{B}_μ . The tensors $V_{\mu\nu}, X^{\mu\nu}, Y_{\mu\nu}$ and $Z^{\mu\nu}$ are all the bilinear functions of B_μ^i and C_μ^i

$$V_{\mu\nu} = -C_\mu^- B_\nu^3 - B_\mu^- C_\nu^3 + C_\mu^- C_\nu^3 + C_\nu^- B_\mu^3 + B_\nu^- C_\mu^3 - C_\nu^- C_\mu^3 , \tag{28}$$

$$X^{\mu\nu} = -C^{3\nu} B^{+\mu} - B^{3\nu} C^{+\mu} + C^{+\mu} C^{3\nu} + C^{+\nu} B^{3\mu} + B^{+\nu} C^{3\mu} - C^{+\nu} C^{3\mu} , \tag{29}$$

$$Y_{\mu\nu} = -C_\mu^- B_\nu^+ - B_\mu^- C_\nu^+ + C_\mu^- C_\nu^+ + C_\nu^- B_\mu^+ + B_\nu^- C_\mu^+ - C_\nu^- C_\mu^+ , \tag{30}$$

$$Z^{\mu\nu} = -C^{-\mu} B^{+\nu} - B^{-\mu} C^{+\nu} + C^{-\mu} C^{+\nu} + C^{-\nu} B^{+\mu} + B^{-\nu} C^{+\mu} - C^{-\nu} C^{+\mu} . \tag{31}$$

From Eq.(25) and Eq.(26), one can obtain the following mass terms in the Lagrangian for the B_μ^3, B_μ^+ and B_μ^- fields

$$\mathcal{L}_{mass3} = -\frac{3}{8}m_{12}m_{21}B_\mu^3B^{3\mu} , \tag{32}$$

$$\mathcal{L}_{mass+} = \frac{3}{16}m_{21}m_{21}B_\mu^+B^{+\mu} , \tag{33}$$

$$\mathcal{L}_{mass-} = \frac{3}{16}m_{12}m_{12}B_\mu^-B^{-\mu} . \tag{34}$$

The standard mass term in a Lagrangian for any massive vector field $V_\mu(x)$ is given by $\mathcal{L}_m = \frac{1}{2}m^2V_\mu V^\mu$. Therefore above equations demonstrate that the B_μ^3, B_μ^+ and B_μ^- fields are all massive provided that

$$m_3^2 = -\frac{3}{4}m_{12}m_{21} , \quad m_+^2 = \frac{3}{8}m_{21}^2 , \quad m_-^2 = \frac{3}{8}m_{12}^2 . \tag{35}$$

It is intriguing that the mass terms of the intermediate vector bosons are unrelated with the diagonal elements of the fermion mass matrix. One can obtain the following interesting formula from Eq.(35)

$$\frac{3}{8}(m_{12} - m_{21})^2 = m_3^2 + m_+^2 + m_-^2 . \tag{36}$$

Thus Eq.(35) and Eq.(36) reveal the connection between the masses of gauge bosons and the fermion mass matrix.

More generally, the fermion mass matrix must not satisfy the Hermitian condition, that is, $\hat{M}^\dagger \neq \hat{M}$. The most general mass term consistent with fermion number conservation has the form

$$-\bar{\psi}_L \hat{M} \psi_R - \bar{\psi}_R \hat{M}^\dagger \psi_L , \tag{37}$$

where \hat{M} is a completely general complex matrix in the isospin space, ψ_L and ψ_R are the “left-handed” and the “right-handed” fields and defined by

$$\psi_L = P_- \psi , \quad \psi_R = P_+ \psi , \tag{38}$$

where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ are the projection operators. In this general case, we can select an interesting ansatz, e.g. $m_{12} = -m_{21} = m$, then we acquire

$$m_3 = \frac{\sqrt{3}}{2}m , \quad m_+ = \frac{\sqrt{3}}{2\sqrt{2}}m , \quad m_- = \frac{\sqrt{3}}{2\sqrt{2}}m . \tag{39}$$

The equation (26) obviously demonstrates that there are constant terms in the lagrangian of Eq.(25), which can be expressed in terms of C_μ^i

$$\begin{aligned}
\mathcal{L}_{const} &= 2g^2(C_\mu^- C_\nu^3 - C_\nu^- C_\mu^3)(C^{+\mu} C^{3\nu} - C^{+\nu} C^{3\mu}) \\
&- g^2(C_\mu^- C_\nu^+ - C_\nu^- C_\mu^+)(C^{-\mu} C^{+\nu} - C^{-\nu} C^{+\mu}) .
\end{aligned} \tag{40}$$

One can easily find that $\mathcal{L}_{const} = 0$. Hence the special fields $C_\mu^k (k = +, -, 3)$ don't affect the energy density of vacuum.

Since the gauge field determined by the Lagrangian of Eq.(2) satisfies the gauge invariance and involves massless vector bosons A_μ^i , it is renormalizable. In our mechanism, it is proved that the gauge invariance of $SU(2) \times U(1)$ is kept in the transformation of Eq.(12); the fermion mass term appears naturally; the masses of intermediate vector bosons originate from the coupling between the gauge boson fields and the fields $C_\mu^i(x)$. Similar to the standard model with the Higgs mechanism, the gauge theory proposed in this letter should be renormalizable either.

As shown by Fadeev and Popov [5], Feynman graphs including at least one loop diagram of the gauge boson field are no longer gauge invariant in Yang-Mills theory. The reason for this failure is the fact that the Feynman rules describing Yang-Mills theory as obtained so far are incomplete. One must take into account the additional contributions of the so-called ghost fields. These fields are described by the additional Lagrangian

$$\mathcal{L}_{ghost} = -\bar{\chi}^k \hat{\mathbf{L}}^\mu (\partial_\mu \delta_{kl} + g \varepsilon_{klm} B_\mu^m) \chi^l, \quad (41)$$

the ghost fields are denoted by χ^k , where $\hat{\mathbf{L}}^\mu$ is an arbitrary linear operator, which appears in the gauge condition $\hat{\mathbf{L}}^\mu B_\mu^k = 0$. Taking the Lorentz gauge $\partial^\mu B_\mu^k = 0$, one get

$$\mathcal{L}_{ghost} = -\bar{\chi}^k \square \chi^k - g \varepsilon_{klm} B_\mu^m \bar{\chi}^k \partial_\mu \chi^l. \quad (42)$$

These demonstrate that there are ghost-boson couplings in Yang-Mills theory. In the lagrangian of Eq.(26), it is easy to find the couplings between the massive gauge boson fields and the fields $C_\mu^k(x)$. Moreover, ghost propagators may not occur as external lines of a Feynman diagram, since they do not correspond to the physical

modes. In this aspect the fields $C_\mu^k(x)$ are the same as the ghost fields. More concise discussion on the relation between the ghost fields and $C_\mu^k(x)$ will be given in forthcoming paper.

For explaining the idea explicitly, we construct the theory under the primary framework of Yang-Mills field. The parity violation and the down quark mixing can not be included in this frame. The question why nature select the massive fermions and the massive $SU(2)$ gauge bosons can not be answered yet.

In this letter, a new mechanism generating the massive intermediate vector bosons in Yang-Mills theory is proposed without introducing the Higgs boson. The relation between the masses of the gauge bosons and the fermion mass matrix is revealed. At the last of this letter, the properties of $C_\mu^k(x)$ are discussed.

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